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Looking up at the GUT/Planck World

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Abstract

The importance of the mass spectroscopy of the superparticles is emphasized. It will be shown that the gauge coupling constants give us information on the GUT-scale mass spectrum once the superparticle masses are known. The gaugino masses will provide us a model-independent test of the grand unification irrespective of the symmetry breaking pattern. The sfermion masses carry the information on the intermediate symmetries, if present. Combining the mass spectrum, we will be able to distinguish among the GUT models, and hopefully supergravity models also.

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1 STATEMENT

I *assume* that the supersymmetry will be discovered at future colliders throughout this presentation. I do not argue that the supersymmetry *must* be discovered. I am not on a mission of the supersymmetry; I am simply trying to convince you that there are lots of interesting physics *after* the discovery of the supersymmetry. Of course, the discovery itself is very exciting. However, it seems even more exciting to me that we will be able to “peek” the physics at the GUT- or Planck-scale once the mass spectrum of the superparticles is known.

The statement I wish to make in this talk is the following simple one.

The spectroscopy of the superparticle masses will provide us a unique tool to verify/exclude/distinguish the GUT models.

It is often stated to be an embarrassment of the supersymmetry that there are more parameters than in the Minimal Standard Model. It is true that I cannot *predict* the masses of the superparticles due to the existence of many parameters. However, I argue that it is the virtue of the supersymmetry that there are many more masses which are to be measured experimentally. In the non-supersymmetric models, the only clues to the GUT-scale physics is the gauge coupling constants and the nucleon decay. On the contrary, there are many mass parameters which can be measured at future colliders in supersymmetric models. These mass parameters carry the information of the GUT or Planck world; whether there is a unification or not, whether there is an intermediate scale, and so on. For this purpose, precise measurements of the superparticle masses are essential, and we have to design the future collider projects and detectors so that such measurements will be possible.

2 WHAT WE LEARNED FROM THE GAUGE COUPLING CONSTANTS

We have seen from the LEP data that the gauge coupling constants do meet at a point if we assume the particle content of the minimal supersymmetric standard model (MSSM). Though it can be still an accident, we are encouraged to search for the supersymmetry at future colliders.

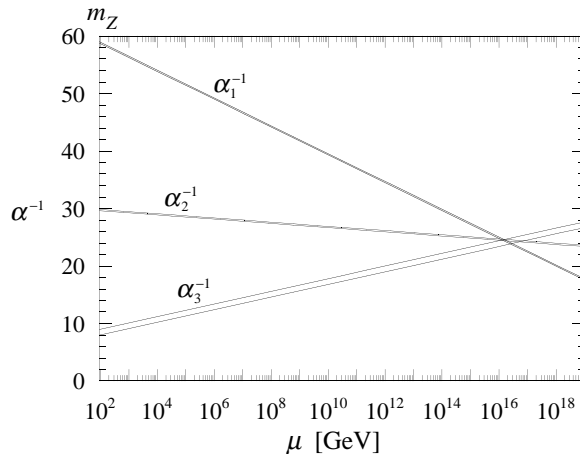


Figure 1: The renormalization group flow of the gauge coupling constants in the Minimal Supersymmetric Standard Model.

It is important to summarize what we learned from the gauge coupling constants measured at LEP. We often hear that the MSSM and $SU(5)$ SUSY-GUT is strongly supported by the LEP data. Indeed, the measured gauge couplings do coincide around the scale $M_X \simeq 2 \times 10^{16}$ GeV with the MSSM particle content (Fig. 1). Note, however, that the gauge coupling constant unification alone does not exclude the existence of light full $SU(5)$ multiplets. There are also many reasons why the minimal $SU(5)$ model may not be the whole story. It has a serious problem concerning the fine-tuning of the parameters to keep Higgs doublets light while making their colored partners superheavy (triplet-doublet splitting). The Yukawa coupling of the light species are known not to obey the relations expected in the minimal model. Furthermore, the cosmic baryon asymmetry which would be generated by the minimal $SU(5)$ model is washed out due to the sphaleron effect since the $B - L$ remains vanishing.

It is interesting that there are $SO(10)$ models with an intermediate scale consistent with the gauge coupling constants measured at LEP. The first example was given by Deshpande *et al.* [1], where the symmetry is broken as $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and then down to the standard model gauge group G_{SM} . One can also build a model with the Pati-Salam symmetry, $SO(10) \rightarrow SU(4)_{PS} \times SU(2)_L \times SU(2)_R \rightarrow G_{SM}$ (Fig. 2) [2]. In both cases, the intermediate scale cannot be determined by

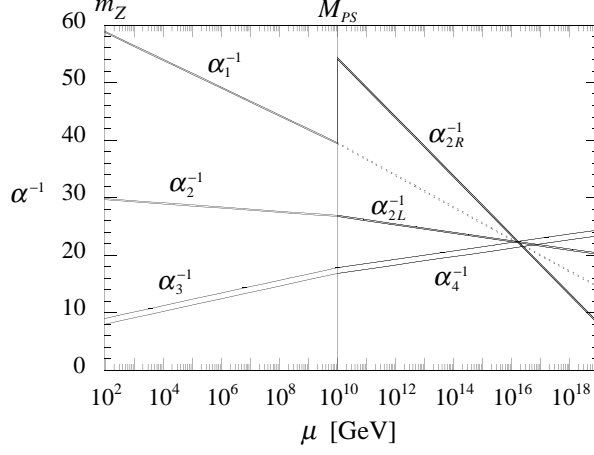


Figure 2: The renormalization group flow of the gauge coupling constants in the model with the chain symmetry breaking, $SO(10) \rightarrow SU(4)_{PS} \times SU(2)_L \times SU(2)_R \rightarrow G_{SM}$ in Ref. [2].

the renormalization group of the gauge coupling constants alone.

Nonetheless, it is interesting to see what we can learn from the gauge coupling constants *assuming* the minimal $SU(5)$ SUSY-GUT. The particle content in the minimal $SU(5)$ model is quite simple, **24**-Higgs Σ to break $SU(5)$ down to G_{SM} , **5** and **5*** Higgses to break electroweak symmetry down to $U(1)_{QED}$, and the matter fields **5*** and **10** for each generations. Of course, the gauge multiplet transforms as **24**. After the breaking of the $SU(5)$ symmetry, the GUT-scale mass spectrum is characterized by only three parameters, mass of the **24**-Higgses M_Σ , mass of the superheavy gauge multiplet M_V , and mass of the colored Higgs M_{HC} . Using the renormalization group equations of the gauge coupling constants, one can *measure* these masses from the LEP data [3]. At the one-loop level of the renormalization group equations, one obtains

$$(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_Z) = \frac{1}{2\pi} \left\{ \frac{12}{5} \ln \frac{M_H}{m_Z} - 2 \frac{m_{SUSY}}{m_Z} \right\}, \quad (1)$$

$$(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z) = \frac{1}{2\pi} \left\{ 12 \ln \frac{M_V^2 M_\Sigma}{m_Z^3} + 8 \ln \frac{m_{SUSY}}{m_Z} \right\}. \quad (2)$$

Here, m_{SUSY} stand for some weighted average of the superparticle masses. Once the mass spectrum of the superparticle is measured, one can determine m_{SUSY} in the above formulae, and then extract the mass of the colored Higgs M_{H_C} , and a combination of M_V and M_Σ from the renormalization group equations. The present bound is shown in Fig. 3, together with the expected limit if the precision of α_s improves by a factor of two. In this way we can obtain an upper bound on M_{H_C} from the gauge coupling constants. On the other hand, super Kamiokande experiment will put a lower bound on the M_{H_C} from the nucleon decay partial life time using the formula (see Ref. [4] for details and notations)

$$\left. \begin{aligned} \tau(p \rightarrow K^+ \bar{\nu}_\mu) &= 0.62 \times 10^{31} \\ \tau(n \rightarrow K^0 \bar{\nu}_\mu) &= 0.35 \times 10^{31} \end{aligned} \right\} \\ \times \left| \frac{0.01 \text{ GeV}^3}{\beta} \frac{M_H}{10^{16} \text{ GeV}} \frac{\sin 2\beta_H}{1 + y^{tK}} \frac{(10 \text{ TeV})^{-1}}{f(q, q) + f(q, l)} \right|^2 \text{ years.} \quad (3)$$

Therefore, the precise determination of the gauge coupling constants will be a crucial step to verify or rule out the minimal $SU(5)$ SUSY-GUT.

3 WHAT WE CAN LEARN FROM THE GAUGINO MASSES

The gaugino masses satisfy a very simple renormalization group equation at the one-loop level,[†]

$$\frac{d}{d\mu} \frac{M_i}{\alpha_i} = 0. \quad (4)$$

Here, M_i is the gaugino mass of the gauge group i , and α_i the corresponding gauge coupling constant.

If the standard model gauge group is unified at some high energy scale, then the boundary condition is that M_i and α_i are common for $SU(3)_C$,

[†]Recently, it was shown by Y. Yamada [5] that this relation is violated at the two-loop level. However, its correction is of the order of a few percents, and irrelevant to the discussion here.

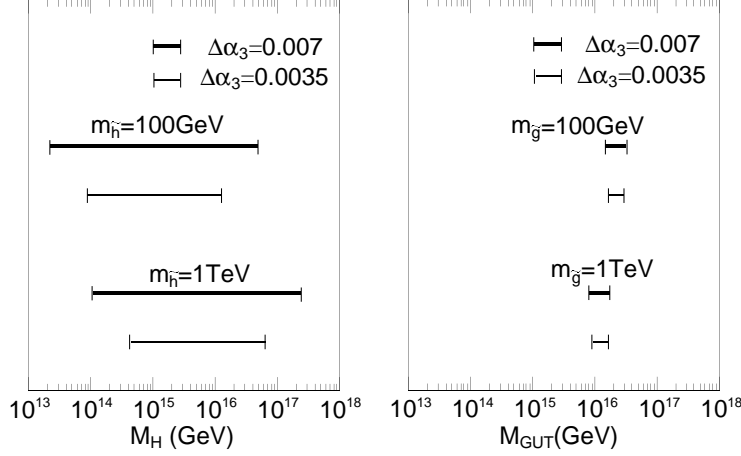


Figure 3: The bounds on the GUT-scale mass spectrum from the gauge coupling constants. The larger error bar corresponds to the present accuracy $\Delta\alpha_s = 0.007$, while the smaller error bar to the improved accuracy $\Delta\alpha_s = 0.0035$. The dependence on the superparticle masses is also shown. The bound on M_{H_C} depends mainly on the higgsino mass $m_{\tilde{h}}$, while that on $M_{GUT} \equiv (M_V^2 M_\Sigma)^{1/3}$ on the gluino mass $m_{\tilde{g}}$.

$SU(2)_L$ and $U(1)_Y$.[‡] Therefore, one expects in $SU(5)$ SUSY-GUT,

$$\frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3}, \quad (5)$$

which is often referred to as the “GUT-relation”. On the other hand, the GUT-relation needs not to hold if the standard model gauge group is not unified, as in some of the superstring models. The simplest example is the flipped- $SU(5)$ model based on the gauge group $SU(5)_{flipped} \times U(1)_{flipped}$ [6], where the $U(1)_{flipped}$ gaugino may have different mass with the $SU(5)$ gaugino at the unification scale,

$$\frac{M_1}{\alpha_1} \neq \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3}. \quad (6)$$

The compactification of extra dimensions in the superstring theory may directly lead to the standard model gauge group. Then all the three M_i/α_i

[‡]Note that the normalization of the $U(1)$ gauge coupling constant may be model dependent. In all known GUT models, however, the standard model is embedded into a group larger than $SU(5)$. I will choose the $SU(5)$ normalization in this talk.

may differ with each other. Even if the gaugino masses are “universal” at the Planck scale, the three gauge coupling constants are different at the Planck scale phenomenologically (see Fig. 1), and hence do not satisfy the GUT-relation. The deviation from the GUT-relation is at the 10 percent level, which can be well distinguished at future e^+e^- colliders. Therefore, the gaugino mass will at least provide an experimental test of the $SU(5)$ SUSY-GUT.

One may suspect that the GUT-relation does not hold when there are intermediate symmetry breakings between the GUT- and the weak-scales, since the $U(1)_Y$ gaugino is a mixture of two or more gauginos which have different masses and different coupling constants. However, one can show that the GUT-relation of the gaugino mass *does* hold even in models with intermediate scales, as far as the standard model gauge group is embedded into a simple group [2]. For example, in the model of Ref. [2], the breaking pattern is $SO(10) \rightarrow SU(4)_{PS} \times SU(2)_L \times SU(2)_R \rightarrow G_{SM}$, and $U(1)_Y$ gaugino is a linear combination of the $SU(4)_{PS}$ and $SU(2)_R$ gauginos. Though the mixing angles depend on the gauge coupling constants of the groups and hence on the particle content above the intermediate scale, the ratio M_1/α_1 remains the same as M_2/α_2 and M_3/α_3 irrespective of the gauge coupling constants and the particle content.

One may also suspect that the threshold corrections at a symmetry breaking scale may ruin the boundary condition that all the gaugino masses should be the same. The threshold corrections may give large effects when logarithms appear in the boundary condition of the renormalization group equations. Indeed, there do appear logarithms in the boundary condition of the gaugino masses depending on the mass spectrum at the symmetry breaking scale. Fortunately, the logarithms completely disappear when we take the ratio M_i/α_i , since there are exactly the same logarithms in the boundary condition of the gauge coupling constants [7]. The remaining threshold corrections are only of the order of the ordinary radiative corrections, and hence of a few percents.

In summary, the gaugino masses will tell us whether the standard model gauge group is unified or not irrespective of the breaking pattern.

4 WHAT WE CAN LEARN FROM THE SFERMION MASSES

I will argue below that the sfermion masses will provide us the most detailed information on the symmetry breaking pattern of the GUT-models [2].

The solution of the one-loop renormalization group equations of the sfermion masses can be given explicitly as follows,

$$m_a^2(\mu) = m_a^2(\mu_0) + \sum_{i=1}^3 \frac{2C_2(R_i^a)}{b_i} (M_i(\mu_0)^2 - M_i(\mu)^2), \quad (7)$$

where i represents the gauge group, a the species of the sfermion, $C_2(R_i^a)$ the second Casimir invariant of the gauge group i for the species a , and b_i 's are the coefficients of the beta-functions defined by $\mu(\partial/\partial\mu)\alpha_i^{-1} = -b_i/(2\pi)$.

Let us discuss the boundary conditions of the sfermion masses at a symmetry breaking scale M_{SB} . Suppose the sfermion species a, b, c, \dots belong to a single multiplet A above M_{SB} . One naively expects a “unification” of the sfermion masses at M_{SB} as

$$m_a^2(M_{SB}) = m_b^2(M_{SB}) = m_c^2(M_{SB}) = \dots = m_A^2(M_{SB}). \quad (8)$$

However, unlike the gauge coupling constants or gaugino masses, there are tree-level effects of so-called D -term contributions [8] which break the “unification” of the sfermion masses. The boundary conditions at the symmetry breaking scale M_{SB} are,

$$m_a^2(M_{SB}) = m_A^2(M_{SB}) + \sum_I g_I^2 Y_I^a Y_I^N \left(|\langle N \rangle|^2 - |\langle \bar{N} \rangle|^2 \right). \quad (9)$$

Here, the symmetry is assumed to be broken by the vacuum expectation values $\langle N \rangle \simeq \langle \bar{N} \rangle$,[§] and Y_I^a are charges of the broken generator I for the species a . The precise values of the D -terms are highly model-dependent, and almost unpredictable. However, the coefficients $Y_I^a Y_I^N$ are determined solely by the group theory. One can also show that the D -terms do not depend on the choice of the Higgs representation up to the overall normalization.

[§]Since a non-vanishing vacuum expectation values of D -terms break supersymmetry, they are of the order of m_{SUSY}^2 , and the difference $\langle N \rangle - \langle \bar{N} \rangle$ is of the order of m_{SUSY}^2/M_{SB} when $M_{SB} \gg m_{SUSY}$.

Therefore, the relative ratios between the D -term contributions to the various sfermions can be predicted for each of the symmetry breaking patterns.

Now we focus on the nearest symmetry breaking above the weak-scale. Assuming that the particle content is the MSSM one below the scale M_{SB} of the nearest gauge symmetry breaking above the weak scale, we know the coefficients of the beta-functions b_i as

$$b_1 = \frac{33}{5}, \quad b_2 = 1, \quad b_3 = -3. \quad (10)$$

The on-shell masses $m_{a,on}^2$ we will measure at future colliders are different from the masses $m_a^2(\mu)$ in the renormalization group equations by the electroweak D -term,

$$m_{a,on}^2 = m_a^2 + M_Z^2(I_a^3 - \sin^2 \theta_W Q) \cos 2\beta, \quad (11)$$

up to possible threshold effects at the weak scale, which are calculable once the mass spectrum is known. If we observe gauginos, squarks, sleptons and Higgs fields and measure their masses and $\tan \beta$, we can obtain the values of sfermion masses $m_a^2(M_{SB})$ from Eqs. (7) and (11). Therefore, we will be able to study the sfermion mass spectrum at the nearest symmetry breaking scale M_{SB} from the TeV-scale experiments alone.

Here, I will discuss only two specific examples. The first one is the $SU(5)$ model, whose nearest symmetry breaking scale is the GUT-scale M_U itself. The boundary conditions are simply

$$m_{\tilde{u}}^2 = m_{\tilde{q}}^2 = m_{\tilde{e}}^2 = m_{\mathbf{10}}^2, \quad (12)$$

$$m_{\tilde{d}}^2 = m_{\tilde{l}}^2 = m_{\mathbf{5}^*}^2, \quad (13)$$

at M_U . This model can be checked in the following manner. First, we already know that the gauge coupling constants are consistent with the expectations in the $SU(5)$ model. Second, once the gaugino masses are measured, we can check whether they satisfy the GUT-relation. At this stage, we still cannot distinguish between the $SU(5)$ model and other GUT-models. However, the sfermion masses provide a highly non-trivial test of the $SU(5)$ model. The masses of \tilde{u} , \tilde{q} and \tilde{e} should meet at a point; and the energy scale should be the same as the scale where the gauge coupling constants meet. The same should be true for \tilde{l} and \tilde{d} . A typical evolution of the sfermion masses is depicted in the Fig. 4.

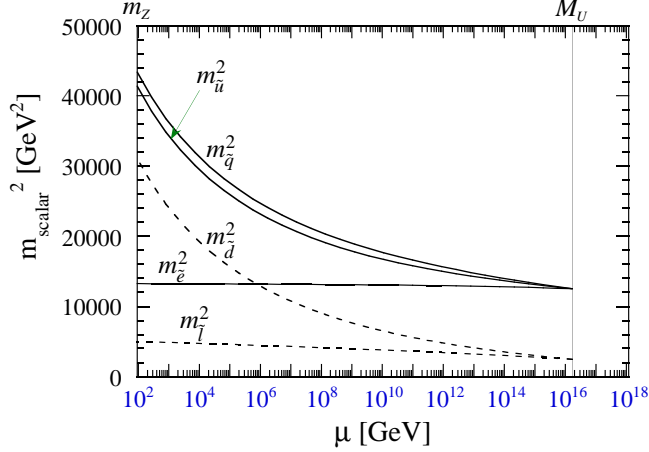


Figure 4: Renormalization group flow of the sfermion masses in the $SU(5)$ SUSY-GUT. The sfermion masses in the same multiplet should meet at the same energy scale M_U where the gauge coupling constants meet.

The second example I discuss here is $SO(10)$ with intermediate Pati-Salam symmetry [2]. Even in this case, the gauge coupling constants meet at the same scale as in the $SU(5)$ model (see Fig. 2), and the gaugino masses satisfy the GUT-relation. On the other hand, sfermions do not have a simple mass spectrum as in the $SU(5)$ model. Instead of Eqs. (12), (13), we have following two relations,

$$m_{\tilde{q}}^2 - m_{\tilde{l}}^2 = m_{\tilde{e}}^2 - m_{\tilde{d}}^2, \quad (14)$$

$$g_{2R}^2(m_{\tilde{q}}^2 - m_{\tilde{l}}^2) = g_4^2(m_{\tilde{u}}^2 - m_{\tilde{d}}^2), \quad (15)$$

at the scale M_{PS} where the Pati-Salam symmetry is broken down to the standard model gauge group. One of the relation should be used to determine M_{PS} , and the other can be used to check the model. A typical evolution of the sfermion masses is depicted in Fig. 5.

5 SUPERGRAVITY

Once the mass spectrum of the superparticles is known, then probably all the theorists will start seriously discussing the dynamics of the supersymmetry

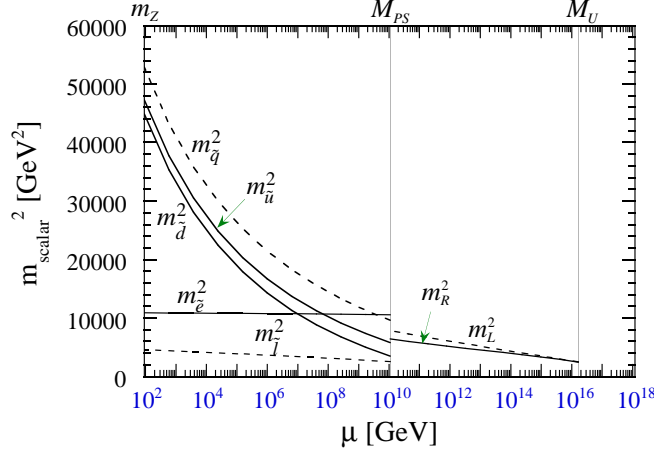


Figure 5: Renormalization group flow of the sfermion masses in the model with intermediate Pati-Salam symmetry. The sfermion masses receive D -term contributions at M_{PS} . They have to satisfy the relations (14) and (15) at M_{PS} .

breaking. Though it is usually believed that the supersymmetry breaking occurs due to a dynamics around the Planck scale, there is no compelling idea how the supersymmetry is broken at present. It may be interesting to recall that there are models of the supersymmetry breaking which predict characteristic patterns of the superparticle masses. The gaugino condensation in the hidden sector generally leads to a baroque spectrum of sfermions, which may generate unacceptably large flavor changing neutral currents. Phenomenologically, models which predict “universal” scalar mass is preferred. The no-scale model [9] predicts that all the sfermion masses vanish at the Planck scale (and hence universal), and the “universal” gaugino mass is the sole origin of the superparticle masses. The dilaton-induced breaking [10] predicts that the “universal” gaugino mass is $\sqrt{3}$ times the “universal” scalar mass.

Probably, the mass spectrum of the superparticles is the only clue to the dynamics around the Planck scale. Since not all the theoretical possibilities are classified at present, we cannot make definite predictions yet. I am expecting the input from the experiments will change this frustrated situation.

6 PROSPECTS AT JLC

In regard of the previous discussions, one has to measure the masses of the superparticles to compare with the predictions of various models. I do not have a definite idea yet how accurately the masses should be measured. For instance, the scalar masses should be measured quite precisely if they are much heavier than the gaugino masses, since only their *splittings* carry the information of the symmetry breaking pattern. If the sfermions are as light as the gauginos, then they do not have to be measured so accurately.

Anyway, the only future project which has potential to measure the superparticle masses at the percent level is the e^+e^- linear collider, such as JLC. It is a matter of luck whether we will be able to find some of the superparticles at an e^+e^- linear collider of $\sqrt{s} = 500$ GeV. But I strongly hope it will turn out to reality so that we can distinguish among the GUT-models in the next 10 years.

Indeed, a detailed analysis has shown that the slepton and LSP masses can be measured at a percent level; also the GUT-relation of M_1 and M_2 can be checked at a similar accuracy [11]. The mass splitting between the right-handed and left-handed slepton depends little on $\tan\beta$, and hence the difference between m_{5*}^2 and m_{10}^2 can be measured very well.

Also, an e^+e^- linear collider with $\sqrt{s} = 500$ GeV can study the top quark threshold region, which will enable us to determine the QCD coupling constant α_s at a very high precision, $\Delta\alpha_s \simeq 0.002$ [12]. As discussed in the section 2, the precise value of α_s is useful to determine the GUT-scale mass spectrum if we assume a specific GUT-model.

One serious question is how we can measure precisely the mass of the colored superparticles. As for the squarks, we will be able to produce them in pairs if we can go up to sufficiently high energy with an e^+e^- machine. The problem is whether we can entangle the cascade decays. For gluino, the situation is worse. We may be able to produce gluino from the squark decay if kinematically allowed, which may allow us to measure the gluino mass. If gluino is heavier than all of the squarks, then we do not have any chance to produce gluino at an e^+e^- collider. The only strategy I can think of is to determine the whole superparticle mass spectrum at an e^+e^- machine first, and then compare the overall SSC/LHC data with the Monte Carlo. Anyway, it seems to me challenging to measure the masses of colored superparticles. Detailed and dedicated studies are needed.

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